Particle Dispersion in a Binary Solid-Liquid Fluidized Bed

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The mixing and segregation behavior of liquid fluidized beds containing two different-sized particle species are described mathematically. This model is based on a mass balance, rather than a volume balance, which has been used by most previous researchers. The model also includes the effects of density gradients that are present in the bed. Particle dispersion coefficients in a binary-solid fluidized bed are evaluated using this model. Further, a correlation that allows the calculation of these particle dispersion coefficients is presented, which describes not only dispersion coefficients calculated using the above model, but also those presented by other researchers. It is applicable over a broad range of variables of practical interest and is based on relevant dimensionless parameters to account for the influence of important particle and fluid properties on the particle dispersion characteristics.

Introduction

The particles in a liquid fluidized bed containing two particle species of different diameters are known to show segregation tendencies. Larger-diameter particles tend to occupy the lower regions of the bed, while smaller particles tend to dominate the upper regions of the bed. Between these two zones of relatively pure components, a mixing or transition region exists. In this zone, there is a gradual variation in the concentration of particles of both species. As a result, the bed void fraction changes in this mixing zone, progressively varying from a minimum value in the lower bed region to a maximum value in the upper bed region. The variation in the bed void fraction leads, in turn, to a variation in the bulk density of the bed and causes a nonconstant pressure drop along the bed's axial dimension.

Characterizing the hydrodynamics of the fluidized bed containing different-sized particles is an important step toward the proper design and scaleup of such processes. Liquid fluidized beds containing multiple-size particles can be encountered in many practical applications. For example, the biosolubilization of coal particles in a fluidized-bed bioreactor leads to the presence of particles which widely differ in size. Because mass transfer and hence reaction rate will be a function of particle size, the correct description of the concentration profile of particle species present in the bed is important.

In the case of the fluidization of monosized particles, the primary mechanism of the transport of solid particles in the bed is dispersion, as pointed out by Carlos and Richardson (1968). This, however, is not the case when two different-sized particles are present in the bed. In this case, particle convection becomes important in the mixing region of the bed due to the velocities of individual particle species. The particles tend to move due to the difference between the fluid interstitial velocity and the particle relative velocity in the two-component mixing region of the fluidized bed. Since the particle relative velocity is a strong function of voidage, the particle velocity will, consequently, depend on the local bed voidage. Mathematically, this can be represented as:

$$U_{p_i} = \frac{U_o}{\epsilon} - U_{r_i} = \frac{U_o}{\epsilon} - U_{t_i} f(\epsilon) \qquad (1)$$

where U_{p_i}, U_{r_i}, and U_{t_i} are the particle velocity relative to the wall, the particle velocity relative to the fluid, and the terminal velocity of particle species i, respectively. U_o is the liquid superficial velocity, ϵ is the local bed void fraction, and $f(\epsilon)$ is an appropriate function of the voidage.

The above equation shows that U_{p_i} can have zero, positive or negative values. The magnitude and direction of U_{p_i} depend on U_{t_i} and the local void fraction, ϵ. In the mixing region of the bed, the particle velocity and the void fraction will be functions of axial position. For large particles, U_{p_i} will always be negative in the mixing zone, but will be equal to zero in its

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monocomponent zone. In other words, a large particle, when displaced from its monocomponent zone, will always show a tendency to move downward, that is, toward its monocomponent zone. On the other hand, a small particle will always show a tendency to move upward in the mixing zone, but will possess a zero velocity in its monocomponent zone. The tendency of different-sized particles to possess different velocities in the same voidage environment is responsible for the segregation behavior of the bed. The particle velocity, therefore, is often termed as the segregation or classification velocity.

Unlike the segregational effects of convection, dispersion has a homogenizing effect tending to disperse various particles uniformly throughout the bed. In the absence of dispersion, one would expect complete segregation of the bed into two monocomponent zones of individual particle species. However, it is important that convective and dispersive effects be included in a mathematical description of the fluidized bed.

Therefore, for modeling purposes, a liquid fluidized bed containing two particle species of the same density, but different diameter, can be considered as consisting of three distinct regions, two of which are monocomponent regions of individual particle species where only the particle dispersion dominates. A transition or mixing zone exists between the two monocomponent regions. In this mixing zone, the two rival mechanisms of segregation and dispersion counterbalance each other, causing the overall flux to equal zero at steady state.

The presence of the two counteracting mechanisms of convection and dispersion in fluidized beds containing binary-size mixture of solid particles was recognized first by Kennedy and Bretton (1966). In the literature, this phenomenon is commonly described as:

$$-D_i\frac{dC_i}{dz}=U_{p_i}C_i \tag{2}$$

where D_i is the dispersion coefficient of the ith particle species and C_i is its concentration.

Notwithstanding minor variations in the definition of U_{p_i}, this still continues to be the most widely used approach to describe the mixing and segregation behavior of liquid or gas-liquid fluidization of binary- or multisize mixture of solid particles (Juma and Richardson, 1983; Matsumoto et al., 1991). For example, Al-Dobbin and Garside (1979), like Kennedy and Bretton (1966), used the correlation of Richardson and Zaki (1954) to define the voidage function, $f(\epsilon)$, for the evaluation of U_{p_i} using Eq. 1. However, the void fraction distribution with bed height, in this case, was computed by assuming perfect bed classification. On the other hand, based on their experimental findings, Juma and Richardson (1983) recommended the following relationship for the evaluation of the particle velocity:

$$U_{p_i}=a_i+b_iz \tag{3}$$

where a_i and b_i are constants and z is the distance along the mixing region. This relationship for U_{p_i} was also adopted by Fan et al. (1987) to represent their experimental data of solid particles mixing and segregation in a gas-liquid-solid fluidized bed.

Based on the results of their experimental investigation, Van Der Meer et al. (1984) proposed a concentration dependence of U_{p_i}. They argued that the classification velocity of particles of species i goes to zero when they are in their own monocomponent zone, but attains the peak value, U_{p_io}, when these particles are in the environment of other particle species. In other words,

$$U_{p_i}=0 \text{ if } C_i=C_{i_m};\quad U_{p_i}=U_{p_io} \text{ if } C_i=0 \tag{4}$$

where C_{i_m} is the monocomponent zone concentration of C_i, which can be, in most cases, calculated using the Richardson and Zaki (1954) correlation. These authors suggested that the particle velocity of species i could be described as a linear function of its own concentration:

$$U_{p_i}=U_{p_io}\left(1-\frac{C_i}{C_{i_m}}\right) \tag{5}$$

Using Eq. 5, Di Felice et al. (1987) reported a good agreement of model predictions with their experimental data.

It is worthwhile to point out that most previous studies basically followed the approach of Kennedy and Bretton (1966) as represented by Eq. 2. Implicit in this formulation is the tacit assumption that there are no density gradients present in the bed. This, however, is not the case. While the bulk density is constant in the monocomponent zones of the bed, it is nonconstant in the mixing zone. This fact is clearly reflected in the presence of a curvature in the pressure gradient profile of the bed. In reality, it is the change in the bulk density of the bed that causes such curvature in the pressure gradient profile of fluidized beds containing two different-sized particle species. This can also be seen very clearly from the following development.

The pressure gradient in a fluidized bed containing a binary mixture of solid particle is given as:

$$-\frac{dp}{dh}=(C_l+C_s)(\rho_p-\rho_f)g=(\rho_b-\rho_f)g \tag{6}$$

Here, C_l and C_s are the volume fractions of the two particle species, ρ_p is the particle density, ρ_f is the fluid density, and ρ_b is the bulk density of the fluidized bed. ρ_f and g are always constants in the above equation. Therefore, only a constant value of ρ_b, usually the case for monosized fluidization, will give a linear pressure gradient profile in the bed. On the other hand, a varying value of bulk density will result in a nonlinear profile of pressure gradient in the binary-sized particle fluidized bed.

In view of above discussion, it is worthwhile to carry out an appropriate mathematical treatment of the problem. This analysis must be based on a mass balance, rather than a volume balance. Unfortunately, most previous studies have used the volume-balance approach. While the mathematical description we present here will provide only minor variations in model predictions from those obtained using the volume balances used previously, it is important that the mathematical treatment of such problems be properly formulated. Thus, the important steps needed to arrive at the correct mathematical model are outlined in the next section.

Model Description

For the particle species i, the continuity requirement is:

$$\frac{d}{dz}(j_i) = 0 \tag{7}$$

which implies the z independence of mass flux, j_i:

$$j_i(z) = \text{constant} \tag{8}$$

A sustained semibatch fluidization at the steady state would require the particle mass fluxes to be zero:

$$j_i(z) = 0 \tag{9}$$

To account for the simultaneous particle transport by dispersion and convection in the mixing region of the bed, we write for the particle mass flux:

$$j_i = D_i \rho_b \frac{d\omega_i}{dz} - U_{p_i} \rho_b \omega_i = 0 \tag{10}$$

where ρ_b is the bulk density and ω_i is the mass fraction of i. Note that the correct expression for the gradient for the mass flux in the present case is the $(d\omega_i/dz)$, instead of (dC_i/dz). This is owing to the presence of density variations in the mixing region of the bed that necessitates using a mass-balance approach, rather than one based on volume balance, followed by prior studies.

Since there is no bulk movement of particles in the fluidized bed, the contribution to the convective flux of the ith particle species in Eq. 10 comes from its velocity only. The upward movement of fluid does not contribute directly to the net movement of solid particles in the bed. This issue has been discussed in detail by Carlos and Richardson (1968) in the context of solids movement in monosized particle fluidization.

Similar to Van der Meer et al. (1984), the following concentration dependence of the particle velocity, U_{p_i}, is defined:

$$U_{p_i} = U_{p_{io}} \left(1 - \frac{\omega_i}{\omega_{i_m}}\right) \tag{11}$$

where ω_{i_m} is the mass fraction of the ith particle species in its monocomponent zone.

Introducing the following dimensionless variables,

$$x = \frac{z}{L}; \quad y_i = \frac{\omega_i}{\omega_{i_m}}; \quad Pe_i = \frac{U_{p_{io}} L}{D_i} \tag{12}$$

Hence, Eq. 10 can now be rewritten as:

$$\frac{dy_i}{dx} - Pe_i (1 - y_i) y_i = 0 \tag{13}$$

If $x = \bar{x}_i$ is the point in the mixing zone where the dimensionless concentration y_i becomes 0.5, the solution of the above equation can be given as:

$$y_i = \left\{\frac{1}{1 + \exp[-Pe_i (x - \bar{x}_i)]}\right\} \tag{14}$$

Therefore, for the large particle species, we have:

$$\omega_l = \left\{\frac{\omega_{l_m}}{1 + \exp[-Pe_l (x - \bar{x}_l)]}\right\} \tag{15}$$

and, for the small particle species, we can write:

$$\omega_s = \left\{\frac{\omega_{s_m}}{1 + \exp[-Pe_s (x - \bar{x}_s)]}\right\} \tag{16}$$

where

$$Pe_l = \left(\frac{U_{p_{lo}} L}{D_l}\right); \quad Pe_s = \left(\frac{U_{p_{so}} L}{D_s}\right) \tag{17}$$

Here, $U_{p_{lo}}$ is the classification velocity of the large particles in the voidage environment of their smaller counterparts. Similarly, $U_{p_{so}}$ is the classification velocity of the small particles in the neighborhood of large ones. Subscripts l and s denote large and small particle species, respectively.

It is important to note that larger particle species in the binary solid fluidized bed will have a negative segregation or classification velocity: that is, larger particles will tend to move down toward the bottom of the bed. As a result, Pe_l will be negative. On the other hand, Pe_s will be positive since the smaller particles, in the mixing zone, tend to move up toward the top of the bed.

This model contains two adjustable parameters, Pe_i and $\bar{x}_i$, for each of the two particle species present in the fluidized bed. Consequently, estimation of four parameters will be required from the experimental pressure-gradient data to evaluate the concentration profile of each particle species in the fluidized bed.

Di Felice et al. (1987) postulated that the concentration of the two particle species would be linearly related. This assumption implies:

$$\frac{\omega_s}{\omega_{s_m}} = \left(1 - \frac{\omega_l}{\omega_{l_m}}\right) \tag{18}$$

Substitution from Eq. 15 gives:

$$\frac{\omega_s}{\omega_{s_m}} = \left\{\frac{1}{1 + \exp[Pe_l (x - \bar{x}_l)]}\right\} \tag{19}$$

Comparison of Eqs. 19 and 16 reveals that the assumption of the linearly-related concentration distribution of the two particle species leads to the following condition:

$$Pe_s = -Pe_l; \quad \bar{x}_s = \bar{x}_l \tag{20}$$

Different signs of Peclet numbers of the two particle species simply confirm the fact that the velocities of the individual species are oppositely directed. The simplified model satisfying the above condition will, henceforth, be called the linearly-

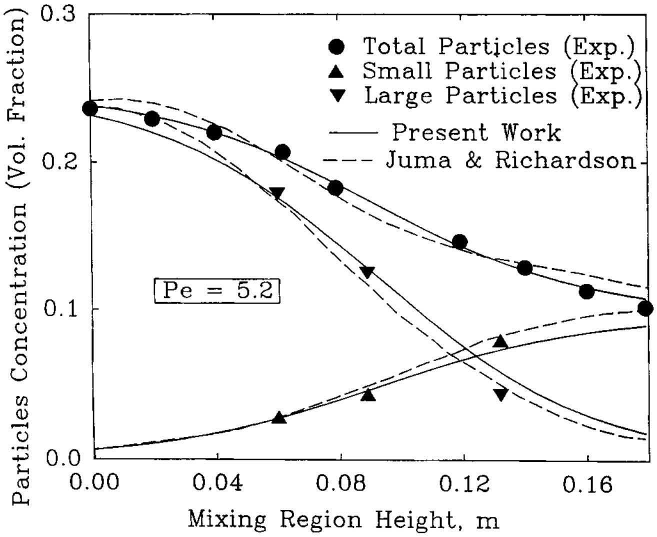

Figure 1. Fluidization mixture of 2 mm and 3 mm glass particles at $U_o = 0.109$ m/s, paraffin oil (Juma and Richardson, 1983).

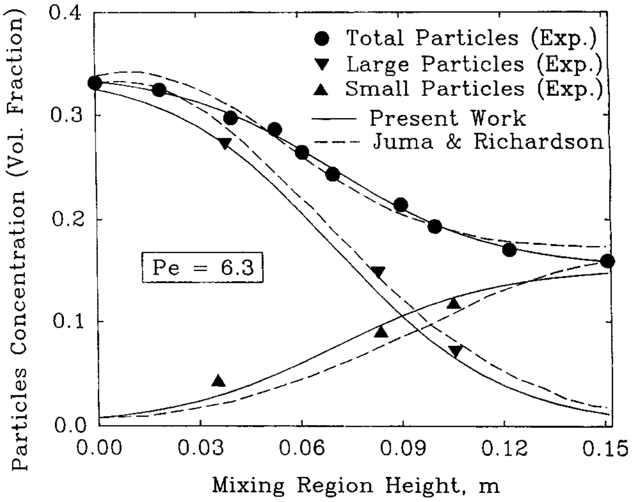

Figure 2. Fluidization mixture of 2 mm and 4 mm glass particles at $U_o = 0.093$ m/s, paraffin oil (Juma and Richardson, 1983).

related concentration distribution model. While the general model described above (Eqs. 15-17) required the evaluation of four parameters, the linearly-related particle concentration distribution model requires only two adjustable parameters: $Pe(=Pe_s=-Pe_l)$ and $\bar{x}(=\bar{x}_s=\bar{x}_l)$ for the evaluation of the concentration profile of the individual particle species.

From a modeling perspective, the linearly-related concentration distribution model with fewer parameters is very attractive. Its usefulness will, however, depend on the validity of its underlying assumption represented by Eq. 20. In this present study, we compare the calculated results of this model with experimental data obtained by Juma and Richardson (1983). In so doing, we also use the model to describe accurately the dispersion and segregation characteristics of the fluidized bed.

Results and Discussion

For comparison, the experimental data of Juma and Richardson (1983) were digitalized and are reproduced here. The linearly-dependent concentration distribution model was used to evaluate the concentration profiles of particle species. The best least-square fit with the experimental data was obtained using a reduced gradient optimization code (Lasdon and Waren, 1986; Edgar and Himmelblau, 1988). The objective function was defined as follows:

$$\theta_1=\sum_{k=1}^{N}[C_{t_e}(x_k)-C_{t_p}(x_k,Pe,\bar{x})]^2 \tag{21}$$

where C_{t_e} is the experimental fractional volumetric concentration of total solid particles, C_{t_p} is the model evaluation, x_k is the kth data point at the location x, and N is the total number of data points. Pe and $\bar{x}$ are the parameters of the model that were varied to obtain the best fit. The mass fractions of the individual species predicted by the model were converted into volume fractions for comparison. The particle concentration in the bottom monocomponent zone of large particles (ω_{l_m}) and the one in the upper monocomponent zone of small particle (ω_{s_m}) were computed using the Richardson-Zaki correlation. In the parameter-fitting procedure, it was found that better results were obtained when a ±5% variation was allowed in the computed values of ω_{i_m}.

It is obvious from Figures 1 and 2 that the model provides a good description of the concentration profiles of the individual as well as the total particles in the bed, utilizing only the experimental pressure-gradient data in the process. It is evident from the comparison with experimental data that the model, in spite of its simplicity, provides a logical explanation of the segregation behavior exhibited by the binary-solid fluidized beds. Moreover, this model is based on a correct mass balance that incorporates the presence of density variations in the mixing region of the bed.

In many practical applications, the particle dispersion coefficients are of primary interest. To evaluate dispersion coefficients, Eq. 17 can be used in conjunction with the estimated value of Peclet number and $U_{p_{i0}}$ computed using Richardson-Zaki correlation. The dispersion coefficients so calculated are shown in Figure 3 for the experimental data of Juma and Richardson. It was found that particle mixing increases with liquid velocity. Also, similar to prior works (Juma and Richardson, 1983), the dispersion coefficients were found to increase with the particle diameter for a particular liquid velocity. Any assumption or simplification suggesting otherwise (for example, Gibilaro et al., 1985) is likely to lead to prediction errors. Using this information and the particle dispersion coefficients calculated as above as well as others reported in the literature, a correlation that allows the particle dispersion coefficients to be estimated was developed. This correlation is presented in the next section.

Correlation for the prediction of particle dispersion coefficients

It is worthwhile at this stage to propose a correlation for the prediction of the particle dispersion coefficients in terms

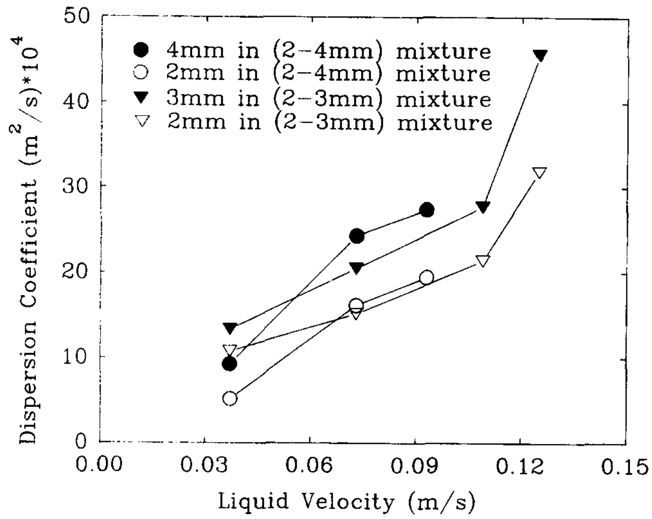

Figure 3. Dependence of particle dispersion coefficients on liquid velocities (paraffin oil).

of relevant dimensionless parameters to ensure its applicability in a wide variety of situations. Previously, Dorgelo et al. (1985) proposed the following correlation for the prediction of the axial particle dispersion coefficient in a liquid fluidized bed:

$$D_i = 0.1 U_o^2 \qquad (22)$$

Here, D_i is the dispersion coefficient with units m^2·s^{-1}, and U_o (m·s^{-1}) is the liquid superficial velocity.

Recently, Kang et al. (1990) proposed the following dependence of the particle dispersion coefficient on the liquid superficial velocity and the minimum fluidization velocity:

$$D_i = 2.97\times10^{-3}(U_o + U_{mf})^{0.802} \qquad (23)$$

Here, the units of variables involved in the above equation are in the MKS system.

These correlations suggest a strong dependence of the particle dispersion coefficient on the liquid superficial velocity. The velocity dependence of the particle mixing was corroborated by the dispersion coefficients obtained in the present work as well. However, the effect of other important fluid and particle properties are not apparent in these correlations, thereby limiting their applicability. For example, Eq. 22 will erroneously predict identical values of the dispersion coefficients for both the large and small particle species in liquid fluidized bed irrespective of their size difference. Moreover, both correlations will unrealistically predict substantial particle mixing in the fluidized bed, even when the liquid velocity is very close to the minimum fluidization velocity.

Therefore, there is a need to develop a suitable correlation based on dimensionless parameters which should be capable of appropriately describing the influence of important system variables on the particle dispersion characteristics. A review of the literature on the subject of fluidized bed and an appropriate dimensional analysis would suggest the use of the following dimensionless parameters:

$$\overline{Pe} = \frac{U_o d_p}{\epsilon D};\quad \frac{U_o}{U_t};\quad Fr = \frac{Re^2}{Ga} = \frac{\rho_f}{\rho_p - \rho_f}\frac{U_o^2}{g d_p} \qquad (24)$$

Here, Fr is the two-phase Froude number, Re is the Reynolds number, and Ga is the Galelio number. Note that the characteristic length used in the definition of $\overline{Pe}$ is the particle diameter, d_p, instead of the length of the mixing zone, L, used in Pe. Since the particle dispersion coefficient depends on its diameter, $\overline{Pe}$, owing to its dependence on the particle diameter, is of primary interest to us as a dimensionless parameter.

To correlate the particle dispersion coefficients in fluidized beds, a wide database, covering a broad spectrum of fluid and solid properties, was considered here. The range of fluid velocities and the important properties of fluid and particle used by different investigators and the values of dispersion coefficients obtained therewith are reported in Table 1. Also, included here are the axial dispersion coefficients reported by Lee and Kim (1990) in gas-solid turbulent fluidized beds. The measurement techniques employed by these workers are also different. While the pressure gradient data of Juma and Rich-

Table 1. Fluid and Solid Properties, Range of Velocities and Particle Dispersion Coefficients Reported by Other Investigators

	Solid Particles		Fluid				
Investigator	Density kg/m^3	Diameter m × 10^3	Density kg/m^3	Viscosity mPa·s	U_o (m/s) × 10^2	D (m^2/s) × 10^4	Symbol
Juma and Richardson (1983)	2,960	0.190	840	10.6	3.7-12.5	5.0-21.0	●
	2,960	0.298	840	10.6	3.7-12.5	13.0-45.0	▲
	2,960	0.370	840	10.6	3.7-9.3	9.0-28.0	■
	2,960	0.214	1,000	1.0	18.2-25.4	8.0-19.0	▼
	2,960	0.291	1,000	1.0	18.2-25.4	21.0-28.0	○
	2,960	0.387	1,000	1.0	18.2-25.4	19.0-23.0	◆
Dorgelo et al. (1985)	2,455	2.3	1,154	9.8	13.0-60.0	8.0-515.0	▽
Carlos and Richardson (1968)	2,480	0.89	1,190	10	7.6-15.1	9.0-32.0	*
Kang et al. (1990)	2,500	0.1	1,000	1.0	3.0-8.0	2.0-4.0	+
	2,500	0.3	1,000	1.0	5.0-15.0	2.0-5.0	–
	2,500	0.6	1,000	1.0	8.0-15.0	3.5-8.5	$
Yutani et al. (1985)	2,500	0.054	1,000	1.0	2.2-3.7	0.70-1.9	△
Muchi et al. (1961)	2,710	0.068	1,000	1.0	1.3-5.4	0.243-3.23	◇
Lee and Kim (1990)	2,500	0.036	0.12	0.0018	20.0-126.0	8.0-81.0	□

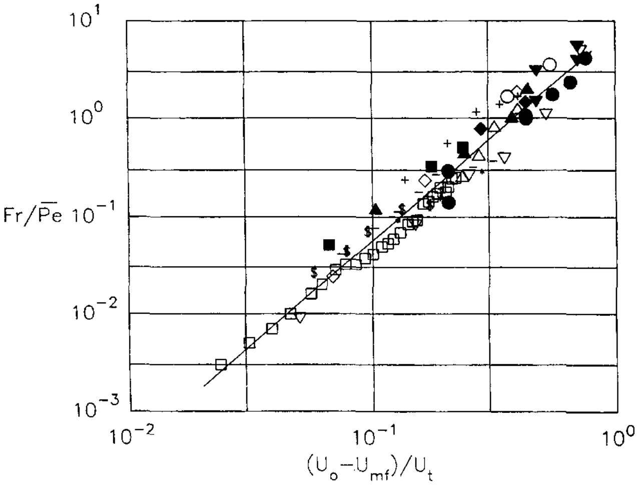

Figure 4. Dependence of $(Fr/\overline{Pe})$ on $(U_o - U_{mf})/U_t$.

ardson (1983) were processed in conjunction with the convection-segregation model presented above to obtain particle mixing coefficients, the steady-state axial transport of heat was used by Lee and Kim (1990) for the evaluation of particle dispersion. On the other hand, Kang et al. (1990) and Yutani et al. (1982) basically employed a pressure relaxation method for quantifying the particle mixing characteristics in liquid fluidized beds. The database of Table 1, therefore, rules out any possibility of error involved with a particular measurement technique influencing the correlation here.

A log-log plot of $(Fr/\overline{Pe})$ vs. $(U_o - U_{mf})/U_t$ is shown in Figure 4. It was found that the dispersion coefficient for both the large and small particles can be best correlated using the following form:

$$\frac{Fr}{\overline{Pe}} = K_1 \left(\frac{U_o - U_{mf}}{U_t} \right)^{K_2} \tag{25}$$

Using the data provided in our modeling efforts and data reported by others, we found K_1 to be equal to 7.9 ± 1.1 and K_2 to be equal to 2.141 ± 0.054. For these values, the correlation coefficient is 0.951 with a 16.4% standard error of prediction. The database of the present correlation covers the following range of dimensionless variables:

$$0.50 \le Re \le 10^3; \quad 0.5 \le \epsilon \le 0.94; \quad 7.0\ 10^{-4} \le Fr \le 1.6$$

Conclusions

The present study accounts for the presence of density gradients in the mass-balance formulation to describe the concentration profiles of individual particle species in a fluidized bed containing binary-sized solid particles. The formulation is based on the existence of two counterbalancing particle transport mechanisms of convection and dispersion in the mixing region of the fluidized bed.

The particle dispersion coefficients are evaluated in this study from the experimental data of Juma and Richardson (1983). Based on the particle mixing data from literature as well as those computed here, a correlation is proposed for the prediction of particle dispersion coefficients in terms of relevant dimensionless parameters. This correlation accounts for important particle and fluid properties and is applicable over a wide range of variables of practical interest.

Acknowledgment

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Notation

a_i = constant in Eq. 3, $m \cdot s^{-1}$
b_i = constant in Eq. 3, s^{-1}
C_i = fractional volumetric concentration of particle species i
C_{t_e} = experimental fractional volumetric concentration of total solid particles
C_{t_p} = predicted fractional volumetric concentration total solid particle
d_p = particle diameter, mm
dp/dh = pressure drop along the height in the bed, $N \cdot m^{-3}$
D_i = dispersion coefficient of particle species i, $m^2 \cdot s^{-1}$
Fr = two-phase Froude number define by Eq. 24
g = acceleration due to gravity, 9.8 $m \cdot s^{-2}$
Ga = Galelio number defined as $[(\rho_p - \rho_f)\rho_f d_p^3 g]/\mu^2$, dimensionless
j_i = mass flux of species i in the mixing region of the bed, $kg \cdot m^{-2} \cdot s^{-1}$
K_1 = coefficient of the correlation in Eq. 25
K_2 = coefficient of the correlation in Eq. 25
L = length of the mixing region, m
n = index of Richardson-Zaki correlation
N = number of data points for C_{te}
Pe = Peclet number defined by Eq. 17, dimensionless
$\overline{Pe}$ = Peclet number defined by Eq. 24, dimensionless
Re = Reynolds number defined as $(U_o \rho_f d_p)/\mu$, dimensionless
U_{mf} = minimum fluidization velocity, $m \cdot s^{-1}$
U_o = liquid superficial velocity, $m \cdot s^{-1}$
U_{p_i} = velocity of particle species i, $m \cdot s^{-1}$
$U_{p_{io}}$ = velocity of a particle of species i in the neighborhood of unlike particles, $m \cdot s^{-1}$
U_{r_i} = relative velocity of particle species i, $m \cdot s^{-1}$
U_{t_i} = terminal velocity of particle species i, $m \cdot s^{-1}$
x = dimensionless mixing region distance
$\bar{x}_i$ = dimensionless distance x for dimensionless concentration, $y_i = 0.5$
y_i = dimensionless variable defined by Eq. 12
z = distance along the mixing region in the bed, m

Greek letters

ϵ = bed voidage, dimensionless
θ_1 = objective function defined by Eq. 21
ρ_b = bed bulk density, $kg \cdot m^{-3}$
ρ_f = fluid density, $kg \cdot m^{-3}$
ρ_p = particle density, $kg \cdot m^{-3}$
ω_i = mass fraction of particle of species i

Subscripts

e = experimental
i = particle of species i
l = large particles
m = monocomponent
p = predicted
s = small particles
t = total particles

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